Mathematical Model of Dengue Fever with and without awareness in Host Population

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Abstract: Dengue is one of the most rapidly spreading diseases in the world. It is transmitted to humans by the bite of infected aedes mosquitoes. In the present paper, the transmission dynamics of dengue disease in the presence and absence of awareness in host population is discussed. It is assumed that some hosts do not interact with the infected mosquitoes as they take different kinds of precautions due to their awareness towards the disease. Some susceptible population is supposed to avoid mosquito bites and some infected hosts are isolated so that they do not transmit the disease. A system of differential equations that models the population dynamics and the associated basic reproduction number are discussed in the paper. Stability analysis is made to determine the dynamical behavior of the system.

Keywords: Dengue; Stability; Awareness; Basic reproduction number

I. INTRODUCTION

Dengue fever is an infectious vector borne disease spreading in tropical and subtropical countries. Dengue is transmitted by aedes mosquitoes. Four serotypes of the dengue viruses DEN 1, DEN 2, DEN 3 and DEN 4 cause the dengue fever. Nowadays, dengue fever is endemic in more than hundred countries. In recent years, the number of dengue cases has been increasing dramatically.

Awareness towards the disease can change the whole dynamics of the transmission of the disease. Due to awareness, people can take different kinds of precautions towards the disease. Avoiding mosquito's bite is the major precaution against dengue fever. Some of the precautions that can be taken are: to keep home, environment and surrounding hygiene, to remove all stagnant water and containers, to cover all containers properly to prevent dengue mosquito breeding there, to wrap all unused plastic tyres, to use mosquito repellents to avoid mosquito bite, to use aerosols and mosquito coils to kill mosquitoes, to wear long sleeve and fully covered clothes, to use mosquitoes net around bed while sleeping etc.

To control the disease effectively, one should understand the dynamics of the disease transmission and take all of the corresponding details into account. Kermack and McKendric contributed on the development of the mathematical theory of epidemics [1]. In the paper, the authors considered three compartments- Susceptible, Infectious and Removal for mathematical formulation of the model. Esteva and Vargas made a study on the transmission of dengue fever with constant human population [2] and variable vector population [3]. Edy and Supriatna developed a transmission model for dengue fever restricting the dynamics for the constant host and vector populations, and reducing the model to two-dimensional planar equations [4]. Different studies have been made to investigate the dengue disease transmission [5 – 9]. The present paper considers SIR model with a fraction of susceptible host and infected host population aware of dengue disease transmission. It is supposed that the populations do not interact with the mosquitoes.

II. FORMULATION OF THE MODEL

To study transmission process of the dengue fever, host population is divided into three compartments susceptible, infected and recovered. People who are healthy and may potentially get infected with dengue virus are considered to be in susceptible compartment, people who are infected with dengue and are able to transmit the disease are considered to be in infected compartment and people who have recovered from dengue disease are considered to be in recovered compartment.

The population of mosquitoes is divided into two compartments only, susceptible and infected compartment: mosquitoes that may potentially become infected with dengue virus are considered to be in susceptible compartment and mosquitoes that are infected with dengue and can transmit the disease are considered to be in infected compartment. The recovered class in the mosquito population does not exist as their infection period ends with their death.

Table 1. Description of state variables

h_n	:	Constant host (human) population size
h_s	:	Number of susceptibles in host population
h_i	:	Number of infectives in host population
h_r	:	Number of removals (recovered) in host population
m_n	:	Vector (mosquito) population size
m_s	:	Number of susceptibles in vector population
m_i	:	Number of infectives in the vector population

In the present model, the fraction p of susceptible host population aware of the disease transmission who use mosquito repellent, mosquitoes net etc. to avoid the mosquito's bite due to awareness is considered. So, the population does not come in contact with the infected mosquitoes. Also, a fraction q of infected hosts is isolated as a result of awareness. Thus, the present paper includes the effect of awareness in the model proposed by Esteva and Vargas [2].

Table 2. The parameters used in the model and their dimensions

p	:	Fraction of susceptible host population aware of dengue transmission, *Dimensionless*
q	:	Fraction of infected host population aware of dengue transmission, *Dimensionless*
h_μ	:	Birth/death rate in the host population, $Time^{-1}$
m_μ	:	Death rate in the vector population, $Time^{-1}$
h_β	:	Transmission probability from vector to host, *Dimensionless*
m_β	:	Transmission probability from host to vector, *Dimensionless*
h_γ	:	Recovery rate in the host population, $Time^{-1}$
b	:	Biting rate of vector, $Time^{-1}$
A	:	Recruitment rate, $Mosquitoes \times Time^{-1}$

The systems of differential equations which describe the present model are;

For human population:

$$\begin{cases} \dfrac{dh_s}{dt} = h_\mu h_n - (1-p)\dfrac{h_\beta b}{h_n} h_s m_i - h_\mu h_s \\ \dfrac{dh_i}{dt} = (1-p)\dfrac{h_\beta b}{h_n} h_s m_i - (h_\mu + h_\gamma) h_i \\ \dfrac{dh_r}{dt} = h_\gamma h_i - h_\mu h_r \end{cases} \qquad (1)$$

For vector population:

$$\begin{cases} \dfrac{dm_s}{dt} = A - (1-q)\dfrac{m_\beta b}{h_n} m_s h_i - m_\mu m_s \\ \dfrac{dm_i}{dt} = (1-q)\dfrac{m_\beta b}{h_n} m_s h_i - m_\mu m_i \end{cases} \qquad (2)$$

Also,

$$\begin{cases} h_s + h_i + h_r = h_n \\ m_s + m_i = \dfrac{A}{m_\mu} \end{cases} \qquad (3)$$

Above equations can be reduced to three equations

$$\begin{cases} \dfrac{dh_s}{dt} = h_\mu h_n - (1-p)\dfrac{h_\beta b}{h_n} h_s m_i - h_\mu h_s \\ \dfrac{dh_i}{dt} = (1-p)\dfrac{h_\beta b}{h_n} h_s m_i - (h_\mu + h_\gamma) h_i \\ \dfrac{dm_i}{dt} = \dfrac{(1-q) m_\beta b}{h_n}\left(\dfrac{A}{m_\mu} - m_i\right) h_i - m_\mu m_i \end{cases} \qquad (4)$$

Introducing the proportions

$$x = \frac{h_s}{h_n}, \quad y = \frac{h_i}{h_n}, \quad z = \frac{m_i}{A/m_\mu} \qquad (5)$$

We obtain,

$$\begin{cases} \dfrac{dx}{dt} = h_\mu (1-x) - \alpha x z \\ \dfrac{dy}{dt} = \alpha x z - \beta y \\ \dfrac{dz}{dt} = \gamma (1-z) y - \delta z \end{cases} \qquad (6)$$

where, $\alpha = (1-p)\dfrac{bh_{\beta}A}{m_{\mu}h_{n}}$, $\beta = h_{\mu} + h_{\gamma}$, $\gamma = (1-q)bm_{\beta}$, $\delta = m_{\mu}$

III. EQUILIBRIUM POINTS AND STABILITY ANALYSIS

A. Disease free equilibrium point

In a disease free situation, $y = 0$, $z = 0$. Hence, from (6), $x = 1$. So, the disease free equilibrium point is (1, 0, 0).

B. Basic Reproduction Number

The basic reproduction number is the expected number of secondary infections produced by an index case in a completely susceptible population. According to the values of the basic reproduction number, disease can persist with $R_0 > 1$ and the disease can die out when $R_0 < 1$.

Using the next generation method, we compute the basic reproduction number R_0, associated with the disease free equilibrium (1, 0, 0). Using last two equations of the system of Equations (6), the non-negative matrix F, of the infection terms and the non-singular matrix V, of the transition terms are given, respectively [10] by:

$$F = \begin{bmatrix} 0 & \alpha \\ \gamma & 0 \end{bmatrix} \text{ and } V = \begin{bmatrix} \beta & 0 \\ 0 & \delta \end{bmatrix}$$

And, therefore

$$FV^{-1} = \begin{bmatrix} 0 & \dfrac{\alpha}{\delta} \\ \dfrac{\gamma}{\beta} & 0 \end{bmatrix}$$

Eigenvalues of FV^{-1} are

$$\pm\sqrt{\frac{\alpha\gamma}{\beta\delta}}$$

Therefore, the basic reproduction number,

$$R_0 = \text{spectral radius of the matrix, } FV^{-1} = \sqrt{\frac{\alpha\gamma}{\beta\delta}}.$$

Proposition 1 (Stability of disease free equilibrium point)

The disease free equilibrium point is asymptotically stable if $R_0 < 1$ *and unstable if* $R_0 > 1$.

Proof: Jacobian matrix of system of equations (6) at the disease free equilibrium point (1, 0, 0) is

$$J = \begin{bmatrix} -h_{\mu} & 0 & -\alpha \\ 0 & -\beta & \alpha \\ 0 & \gamma & -\delta \end{bmatrix}$$

Using the matrix J, we find the following characteristic equation:

$$(h_\mu+\lambda)[\lambda^2+(\beta+\delta)\lambda+\beta\delta(1-R_0^{\ 2})]=0 \qquad (7)$$

From characteristic equation (7), it is observed that first eigenvalue is $-h_\mu$ which is negative. Next,

$$\lambda^2+(\beta+\delta)\lambda+\beta\delta(1-R_0^{\ 2})=0 \qquad (8)$$

The two conditions of Routh Hurwitz Criteria, for local asymptotical stability of second order characteristic polynomial $\lambda^2+a_1\lambda+a_2=0$ are

i. $a_1>0$
ii. $a_2>0$

We have, $a_1=\beta+\delta=h_\mu+h_\gamma+m_\mu$ which is always positive. Also, $a_2=\beta\delta(1-R_0^{\ 2})$.

Here, $\beta\delta$ is positive. And $1-R_0^{\ 2}>0$ if $R_0<1$. So, $a_2=\beta\delta(1-R_0^{\ 2})>0$ if $R_0<1$.

Hence, if $R_0<1$ the eigenvalues will have negative real parts and the disease free equilibrium point becomes asymptotically stable. If $R_0>1$, the two eigenvalues of equation (8) are one negative and one positive real number. So, the disease free equilibrium point becomes unstable if $R_0>1$. □

Proposition 2 (Existence of the Endemic equilibrium point)

The Endemic equilibrium point of the system of equations (6) exists if $R_0>1$.

Proof: Solving the system of equations (6), the equilibrium points found are (1, 0, 0) and (x^e, y^e, z^e) where $(x^e,y^e,z^e)=\left(\dfrac{h_\mu\gamma+\beta\delta}{\gamma(h_\mu+\alpha)},\dfrac{h_\mu(\alpha\gamma-\beta\delta)}{\beta\gamma(h_\mu+\alpha)},\dfrac{h_\mu(\alpha\gamma-\beta\delta)}{\alpha(h_\mu\gamma+\beta\delta)}\right)$

The first point is disease free equilibrium point. The second point becomes endemic equilibrium point and exists if $\alpha\gamma-\beta\delta>0$. Which implies that $\dfrac{\alpha\gamma}{\beta\delta}>1$ and then $R_0>1$. Hence, the endemic equilibrium point exists if $R_0>1$. □

IV. NUMERICAL RESULTS AND DISCUSSION

In the present paper we explored the effect of awareness in the transmission of dengue disease taking different values of awareness parameters *p* and *q*. For the value *p* = 0, *q* = 0 present model changes to the model proposed by Esteva and Vargas [2].

To illustrate the dynamics of the dengue disease, the different values of the awareness fraction *p* and *q* used are given in Table 3.

Table 3. Parameters and their values used

Parameters	h_μ	m_μ	h_β	m_β	h_γ	b	h_n	A	p,q
values	0.0000457/day	0.14286/day	0.75	0.75	0.14286/day	1/3	10000	5000	Variable
references	[2]	[7]	[6]	[6]	[2]	[7]	[2]	[2]	---

Figure 1 is drawn for *p* = 0 = *q* (no awareness). Figures 2 and 3 (with awareness) are drawn for *p* = 0.5, *q* = 0.7 and *p* = 0.8, *q* = 0.9 and compared with the Figure 1. Figures 2 and 3 illustrate the effect of awareness in the transmission of the dengue epidemic showing that there is small number of people infected of disease and large number of people remained susceptible when compared with Figure 1. The size of infected mosquito population is seen smaller in Figure 2 and 3 than in Figure 1.

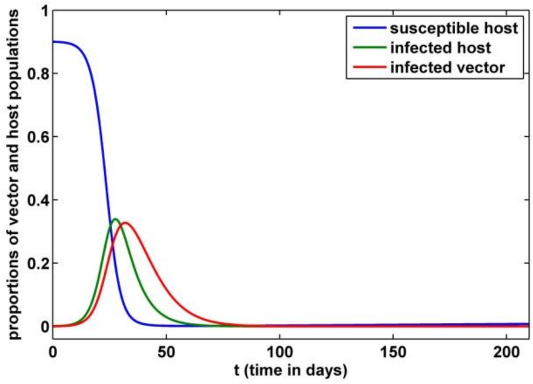

Figure 1. *Dynamics of host and vector population with p = 0 = q.*

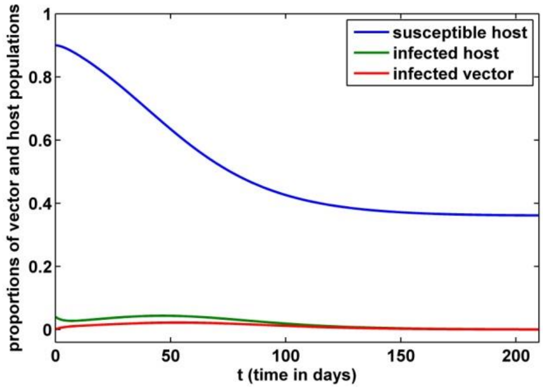

Figure 2. *Dynamics of host and vector population with p = 0.50, q = 0.70.*

The infection rate is observed higher when there is no awareness (Figure 1) than when there is awareness in people (Figure 2 and Figure 3). Also, the rate of infection is seen decreasing with the increasing values of awareness parameters (Figure 2 and Figure 3). Figure 3 shows that when there is sufficient number of people aware of the disease transmission, there is very less number of host and vector infected of the disease. Thus, increasing awareness in people can decrease the infection rate and consequently can decrease the number of infected population.

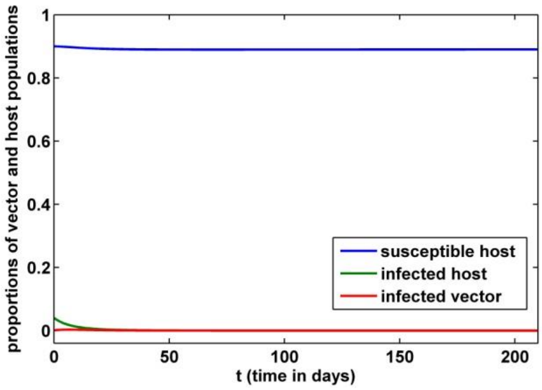

Figure 3. *Dynamics of host and vector population with p = 0.80, q = 0.90.*

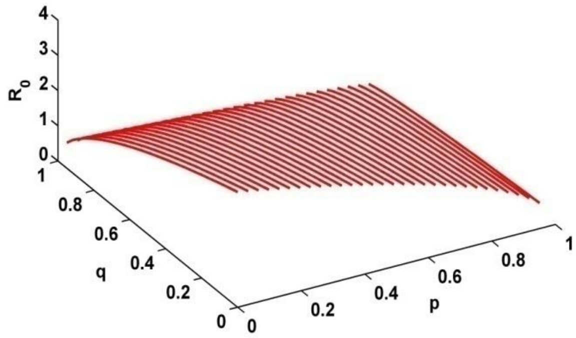

Figure 4. *Basic reproduction number with different values of awareness terms p and q.*

For p = 0 = q, the value of basic reproduction number, $R_0 = 3.27$ and for p = 0.8, q = 0.9, the value of basic reproduction number, $R_0 = 0.46$. Thus, the value of basic reproduction number is less when there is no awareness and more in the presence of awareness in host population. Also, Figure 4 shows that the value of basic reproduction number approaches unity and becomes less than unity for sufficiently higher values of awareness parameters p and q.

V. Conclusion

A dengue model was studied by including the impact of the awareness parameters in the transmission dynamics of dengue fever. An analysis of the disease transmission was made with and without the presence of awareness in the host population. A fraction of susceptible host population is supposed not to come in contact with mosquitoes. The population is supposed to be aware of the disease transmission and they are supposed to take all possible precautions such as using mosquito repellent, mosquito net etc so that they can avoid the bite of mosquitoes. Also, a fraction of infected host population is supposed not to transmit the disease as the population is isolated due to awareness.

Small values of basic reproduction number were appeared in the presence of higher levels of awareness and large value of basic reproduction number was appeared in the absence of awareness in host population. Hence, the spread of the disease comes under control with the increase in awareness in host population.

For higher level of awareness, the disease is seen to affect less number of people and mosquitoes. Large number of people is seen to be affected from the disease when there is no awareness in the host population. So, the present study suggests that with the increase in the awareness in host population, remarkable susceptible host population size can be saved from being infected. So, spread of awareness about the disease transmission plays an important role in controlling the dengue disease transmission.

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