

# Establishing Cost of Manufacturing Parameters for Cylindrical Features with Geometric Tolerances

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**Abstract:** Cost of manufacturing is an essential factor for estimating cost of a product and establishing optimum manufacturing strategies. One of the most significant factors that influence cost of manufacturing is tolerance. Traditionally, cost formulations were established based on a single size tolerance parameter. However, with geometric tolerances, as is the case for virtually all part /assembly tolerancing processes, it may not be always feasible to create one single tolerance parameter that could be used for estimating the cost functions. It has been shown in earlier studies that set of generic deviation parameters could be used to represent the geometric tolerances as per ASME Y14.5 specifications and these deviation parameters could be used to develop cost formulations at various geometric tolerancing conditions. In this paper, the authors present establishment of cost coefficients for machining cylindrical features subjected to geometric tolerances, using turning operations. Results from the manufacturing experiments carried out at various combinations of speed, feed and depth of cut on cylindrical samples (external cylindrical feature) of specified nominal size and tolerances are presented. The deviations of the machined surfaces were measured for establishing the virtual condition boundary (VCB) of the cylindrical features and subsequently, computation of the cost coefficients have been carried out. The results are presented along with discussion on the applicability and limitations of these cost models.

**Keywords:** Manufacturing, GD&T, CAD, Metrology, Materials & Processes

## I. INTRODUCTION

For the manufacturing industry, one of the fundamental challenges is to survive the global competition and maintain a sustainable manufacturing practice. Typically lean and sustainable operations techniques are used to deal with these issues. Cost of manufacturing is a major factor in all these and in lean parlance, reduction of waste (and cost) without affecting product functionality and quality is fundamental and desirable. Thus, estimation of cost of manufacturing a part before the actual production takes place, is essential. This helps in optimizing alternate manufacturing strategies to control the cost of a product. Of the many factors that affect cost of manufacturing, tolerance is probably the single most important parameter as the tolerance values dictate the cost: tighter (smaller) the tolerance, higher the cost of manufacturing the part and vice-versa. Traditionally, an inverse power functions (or similar variations like exponential decay, etc.) of a single tolerance (plus-minus size tolerance) parameter have been used in most cost-tolerance formulations [1-3]. Essentially, these cost of manufacturing formulations are monotonically decreasing functions of a single variable. However, from manufacturing and functionality point of view, one size tolerance is not always sufficient to represent/control all aspects of manufacturing, assemblability and functionality of parts. Geometric tolerances are often used as per ASME Y14.5 [4] along with size tolerances to meet the quality standards and fulfill functional requirements. For example, [5], for the part as shown in Fig. 1, on top of the traditional size tolerances, a positional tolerance is specified that uses multiple datum references and also it includes a material control specifier, called the maximum material condition (MMC),

indicated by the symbol **m**, that allows one to produce the part with additional bonus tolerances (reduced cost of manufacturing).

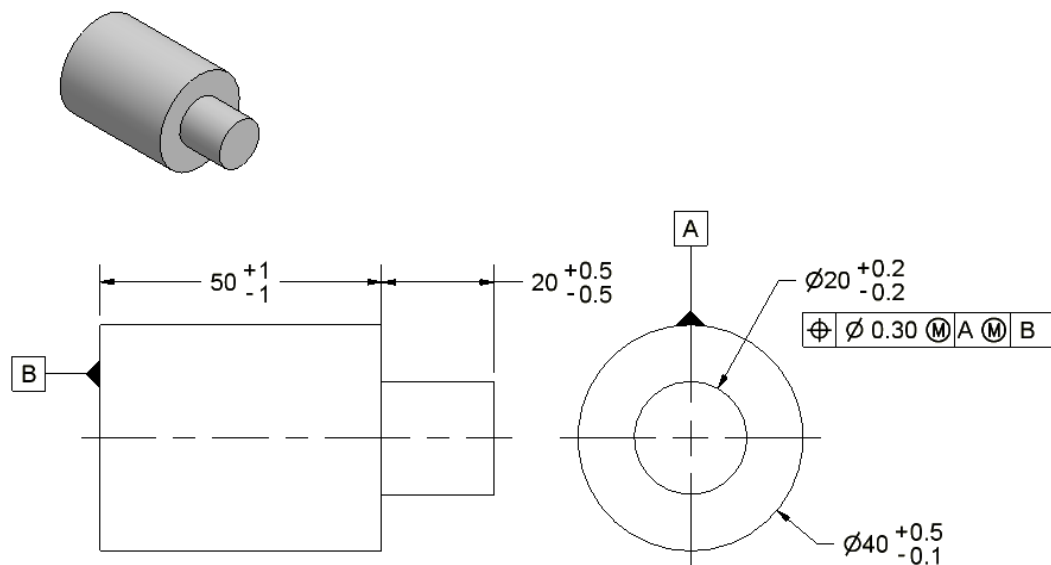


Figure 1. Geometric tolerance specified on a cylindrical feature [5]

In cases like this, it is very difficult (in most cases impossible / impractical) to establish a single representative tolerance parameter that can be used for estimating cost of manufacturing.

In order to take the prescribed tolerance conditions into account for cost estimates, one needs to establish a virtual condition boundary (VCB) within which the actual mating envelope (AME) of the cylindrical surface should remain. The above example is a relatively simple case of a single part with one geometric tolerance; even then, establishing an explicit single tolerance parameter that could correctly represent the behavior of the cylindrical feature is difficult. We need a more generic cost formulation that could take into account possible variation vis-à-vis the geometric tolerances. Unfortunately, there is no such universal formulation available and in order to circumvent these difficulties, the cost functions have to be extended from the one-dimensional tolerance domain to a generalized deviation domain so that the total effect of geometric tolerances prescribed for all features of a part could be taken into account, as has been shown in [6 - 8].

In this paper, the authors present results from machining experiments that were conducted to establish cost parameters for the deviation-based cost of manufacturing formulations for cylindrical features for turning operations at various combination of speed, feed and depth of cut. After the machining operations were carried out, the surface deviations were measured at pre-defined points on the surface using a digital profilometer. The deviation data were subsequently analyzed to develop equations of best-fitted circles and cylinders which would then be used to compute the deviations. The VCB of the cylindrical feature were developed by computing the equation of cylinders to enclose the deviations of the cylindrical surface. Cost coefficients are then computed from these deviation parameters.

A brief description of the deviation-based cost of manufacturing formulations is presented next, followed by the experimental setup, machining parameters, measured data, computed results of the study and conclusion.

## II. COST OF MANUFACTURING FORMULATIONS

The deviation-based cost of manufacturing formulations are based on feature variations represented in terms of small deviations at strategic points on a feature called small deviation torsors (SDT) [9, 10].

These SDTs have six components (three linear, three rotational) corresponding to the six degrees of freedom (DOF) associated with a feature. Although these SDTs look like 6-component vectors, they are not true vectors as three of the components are linear and the other three are rotational and they follow a different transformation rule [9].

When the displacements are small, SDT could effectively represent the behavior of the feature. It has been established [11, 12] that geometric tolerances specified as per ASME Y14.5 could be mapped to these generic deviation parameters through a series of explicit and/or implicit mathematical relations. These mapping relations become a set of constraints that restrict the domain of the deviation parameters in the deviation space. Cost functions could then be defined in terms of these deviations. Details of procedure for establishment of these generic cost functions have been shown in [5]. The generic cost functions are of the following equation (1) form:

$$C(d) = C_1(d_1) * C_2(d_2) * \dots * C_6(d_6) \quad (1)$$

Where  $d = (d_1, d_2, d_3, d_4, d_5, d_6) = (\theta_x, \theta_y, \theta_z, \delta_x, \delta_y, \delta_z)$  are the deviation parameters of the feature and  $C_1, C_2$ , etc are cost functions that depend on  $d_1, d_2$ , etc.

Depending on the nature/type of the feature, some of the six functions will be constants (invariants) and could be eliminated, for example, for some  $j$ , if the cost is invariant, we could use  $C(d_j) \equiv 1$ . This will correspond to the deviation parameters that are invariants of the feature. An example of such cost function for a planar-circular feature has been shown in [5].

#### A. Cost Formulation for Cylindrical Features

In this present work, the authors explore the cost-deviation formulations for a cylindrical feature that has been machined using turning operations. It has been established in earlier studies that mapping deviation parameters to GD&T specifications as per ASME Y14.5 for cylindrical features is complex and often it doesn't have a compact analytical solution [12]. However, for specific cases of tolerances, procedure to derive closed form tolerance zones could be established. In this case, a positional tolerance specified at MMC for the larger cylindrical feature (Fig. 2) is considered. After the features have been machined, measurements are carried out at three elevations as shown in Fig. 3. At each elevation, measurements are taken for the surface deviations at the specified 12 points (Fig. 3) uniformly distributed on the surface using a profilometer. Thus, each cylindrical sample will have 36 points on the three planes for establishing the VCB.

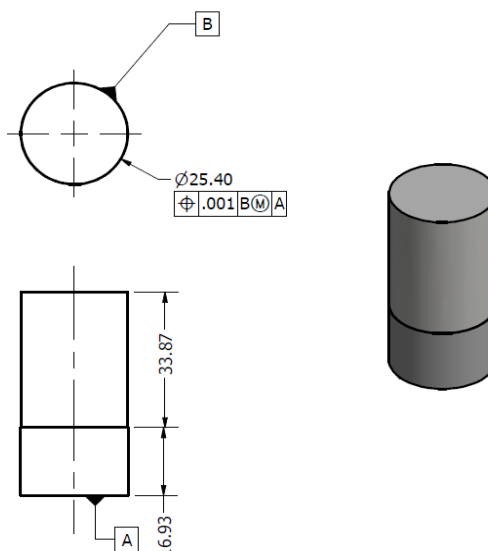


Figure 2. Cylindrical feature for turning

The twelve data points on each circular section with the radial mean deviation  $\delta_{rj}$ ,  $j=1$  to 12, would give the coordinates of the deviated point as  $((R + \delta_{rj})\cos\theta_j, (R + \delta_{rj})\sin\theta_j)$ ,  $\theta_j = 2\pi j/12$

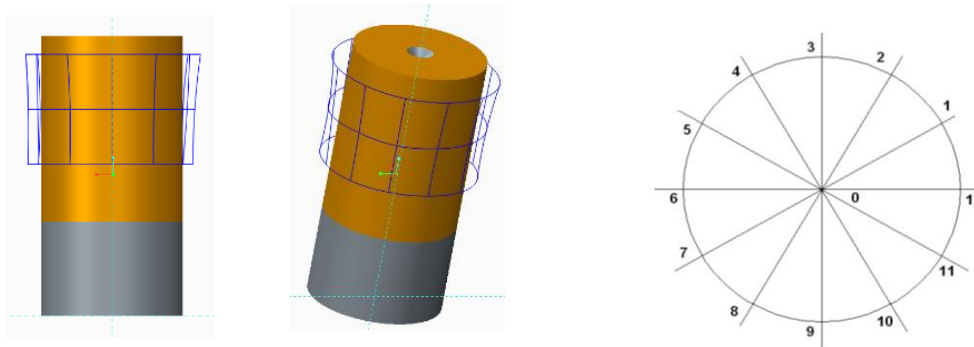


Figure 3. Cylindrical surface deviation measurement points

Using these twelve points, a best fitted circle is then generated applying the minimization of SRSS error to find three parameters  $(d_{xi}, d_{yi}, r_i)$ ,  $i=1$  to 3, where  $d_{xi}$  are deviations of the center and  $r$  is the radius of the circle that represent the deformed shape of the cylindrical feature. Excel Solver add-in is used to perform the Non-linear constrained minimizations. In the next phase, the AME of the cylinder is established as a theoretical cylinder that tangentially encloses all the three circles derived above. These computations are performed using the algorithm detailed below:

Given three circles with parameters: center  $(a_i, b_i)$ , radius  $r_i$ ,  $i=1,2,3$

Find the circle with parameters: center  $(p, q)$ , radius  $r$  of the circle  $(x-p)^2 + (y-q)^2 - r^2 = 0$  such that it encloses the 3 specified circles.

First consider the first two circles and calculate as below:

$$d = ((a_1 - a_2)^2 + (b_1 - b_2)^2)^{0.5} \quad \text{'distance between the centers}$$

if  $d > 0$  then 'the two circles are not concentric

$$\theta = (r_2 - r_1 + d) / (2d) \quad \text{'parameter to locate point on the line connecting the centers}$$

'New circle has parameters: center  $(a', b')$ , radius  $r'$  calculated by

$$a' = a_1 + \theta * (a_2 - a_1)$$

$$b' = b_1 + \theta * (b_2 - b_1)$$

$$r' = (r_1 + r_2 + d) / 2$$

else 'these two are concentric circles, take the larger circle

$$a' = a_1$$

$$b' = b_1$$

$$r' = \max(r_1, r_2)$$

end if

Repeat the above process with the 3<sup>rd</sup> circle and the new circle parameters  $(a', b', r')$  to get the final circle parameters as: center  $(\Delta_x = a', \Delta_y = b')$  and radius  $(R_0 + \Delta R) = r' \rightarrow \Delta R = r' - R_0$ , where  $R_0$  is the nominal radius of the cylinder.

Thus, the required deviation parameters of each of the cylindrical features are represented by  $(\Delta_x, \Delta_y, \Delta R)$ . These three deviation parameters, equation 2, represent the VCB of the cylindrical feature (Fig. 4) with positional tolerance as specified:

$$VCB \equiv \{ (x - \Delta_x)^2 + (y - \Delta_y)^2 - (R_0 + \Delta R)^2 = 0, \forall z \in (0, H) \} \quad (2)$$

H = height of the cylinder.

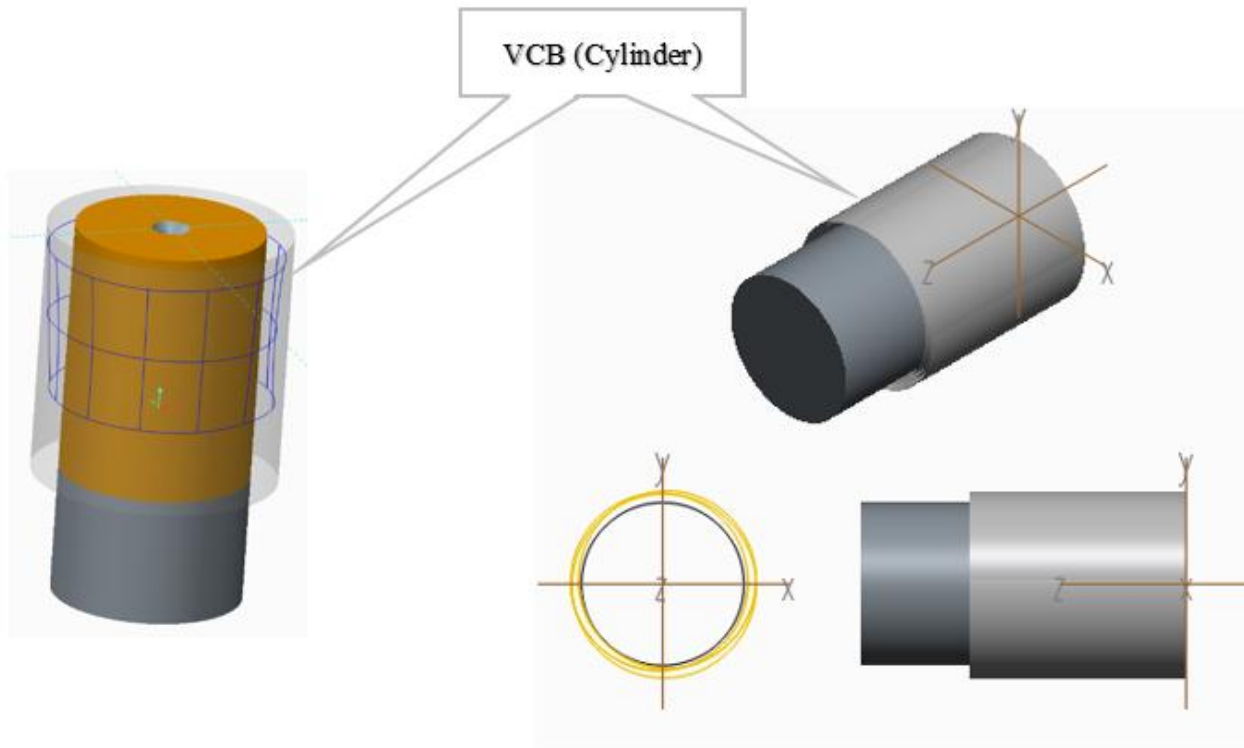


Figure 4. VCB of the Cylindrical Feature

The maximum deviation of the cylindrical feature is given by equation (3):

$$\delta_{max} = \Delta R + \sqrt{(\Delta_x^2 + \Delta_y^2)} \quad (3)$$

In order to establish the link between the cost of manufacturing and the deviation parameters, basic cost formulation for machining cylindrical features derived in our earlier work [8] is used. The cost function is given by the equation (4)

$$C_{cylindrical} = \frac{2\pi R H K_p}{(\epsilon_0 + \delta_{max})} \quad (4)$$

Where  $K_p$  is the cost coefficient,  $\epsilon_0$  is a small parameter used to eliminate singularity at zero deviation/tolerance.

The shape of this cost function is symmetric about the vertical (cylinder) axis. A plot of the above cost function with assumed coefficients are show in Fig. 5.

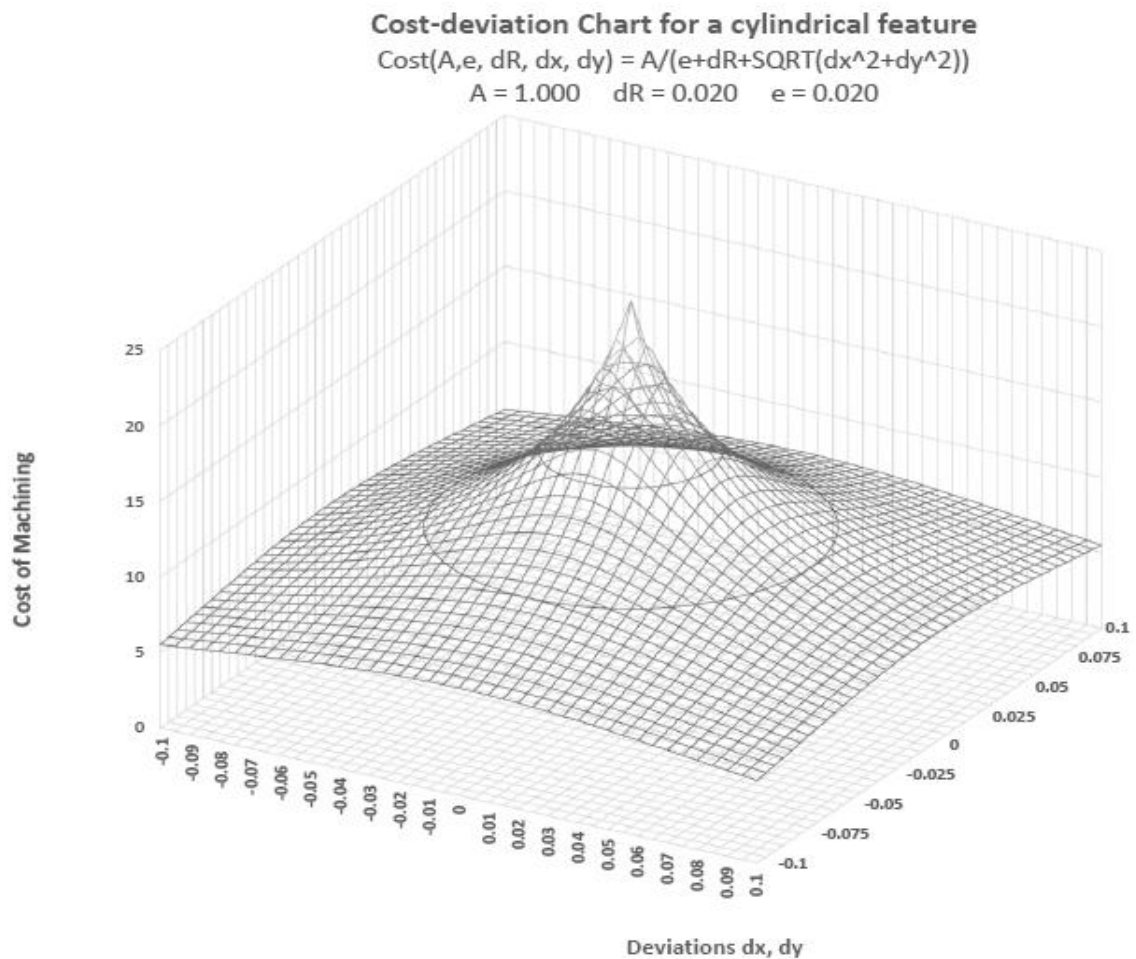


Figure 5. Cost of Manufacturing as Function of Deviation Parameters

Cost of machining a perfect cylindrical shape with zero tolerance ( $\delta_x = \delta_y = \delta_r = 0$ ) will be impossible and so the cost function has a singularity at that point, however, for practical usage, the singularity is avoided by adding a small constant  $\epsilon_0$  as shown in equation (4), making the cost at zero-tolerance finite but as high as desired. The cost coefficients  $K_p$  is computed from equation (4) as given in equation (5) below:

$$K_p = \frac{(\epsilon_0 + \delta_{max})C_{cylindrical}}{(2\pi RH)} \quad (5)$$

### III. MANUFACTURING SETUP AND PARAMETERS

Plain carbon steel rod (material: SAE1018 with Brinell Hardness Number (BHN) in the range of 125-175) of diameter one inch (25.4 mm) was used for the work samples and twenty-four (24) 2-inch (50.8 mm) length sample pieces were cut from the rod (Fig. 6).



Figure 6. Sample Work Pieces

Harrison M 300 Lathe machine was used for the machining operations and Taylor/Hobson Precision made Surtronic25 was used for measuring the deviations of the machined surfaces. For the turning operations, Valentine PTFE uncoated carbide cutting tool was used and cutting speeds and feeds were selected from [13]. However, the optimum feed and speed were not always available on the Harrison lathe and in such cases the closest available speed and feed rates were used. Cutting parameters and actual observed cutting times are shown in Table 1.

After the parts were machined, the deviations of the circular feature of each part were measured at the 3 X 12 predefined positions (Fig. 3) using the Profilometer. The part was mounted on a graduated metal base and measurements were taken at each of the 12 angular positions.

Table 1. Machining Parameters

Samp#	Feed (mm/rev)	Speed (rpm)	Depth of cut (mm)	Op.Time (sec)
1	0.4572	800	0.254	19
2	0.4572	800	0.254	19
3	0.2286	800	0.508	17
4	0.2286	800	0.508	17
5	0.2286	1200	0.254	11
6	0.2286	1200	0.254	11
7	0.2286	1200	0.508	12
8	0.2286	1200	0.508	12
9	0.1016	1200	0.254	22
10	0.1016	1200	0.254	22

Table I: Continued...

11	0.1016	1200	0.508	22
12	0.1016	1200	0.508	22
13	0.0508	1200	0.254	45
14	0.0508	1200	0.254	45
15	0.0508	1200	0.508	45
16	0.0508	1200	0.508	45
17	0.91694	1200	0.254	11
18	0.91694	1200	0.254	11
19	0.91694	1200	0.508	11
20	0.91694	1200	0.508	11
21	0.91694	800	0.254	19
22	0.91694	800	0.508	19
23	0.91694	800	0.254	19
24	0.91694	800	0.508	19

#### IV. ANALYSIS OF DATA TO ESTABLISH THE COST COEFFICIENTS

The measured surface deviations are shown in Appendix-1 and the procedure described above are used along with equations (3), (4), (5) to compute the deviations and cost coefficients. The computed values are shown in Table 2.

Table 2. Computed Deviation Parameters and Cost Coefficients

Sample	Deviation parameters (mm)					VCB Ø (mm)	Cost Coeff Kp
	$\delta X$	$\delta Y$	$R+\delta R$	$\delta R$	$\delta Max$		
1	-3.733E-04	4.016E-04	1.270E+01	3.055E-03	3.603E-03	25.4061	1.689E-05
2	4.355E-04	6.309E-04	1.270E+01	4.062E-03	4.828E-03	25.4081	2.263E-05
3	5.485E-06	2.654E-04	1.270E+01	3.870E-03	4.136E-03	25.4077	1.734E-05
4	2.850E-04	1.945E-04	1.270E+01	4.661E-03	5.006E-03	25.4093	2.100E-05
5	-4.802E-06	-7.696E-05	1.270E+01	2.954E-03	3.031E-03	25.4059	8.225E-06
6	6.342E-05	1.236E-04	1.270E+01	3.430E-03	3.569E-03	25.4069	9.685E-06
7	-3.804E-05	-1.650E-04	1.270E+01	2.934E-03	3.104E-03	25.4059	9.187E-06

Table: II Continued...



8	8.158E-05	-2.681E-04	1.270E+01	2.671E-03	2.951E-03	25.4053	8.735E-06
9	3.004E-04	8.604E-05	1.270E+01	3.463E-03	3.775E-03	25.4069	2.049E-05
10	-2.109E-04	7.852E-05	1.270E+01	3.183E-03	3.408E-03	25.4064	1.849E-05
11	5.133E-05	-3.665E-04	1.270E+01	3.042E-03	3.412E-03	25.4061	1.852E-05
12	-3.270E-05	-4.016E-04	1.270E+01	2.942E-03	3.345E-03	25.4059	1.816E-05
13	-1.807E-04	-1.259E-04	1.270E+01	2.012E-03	2.233E-03	25.4040	2.478E-05
14	1.019E-04	2.792E-04	1.270E+01	1.615E-03	1.912E-03	25.4032	2.123E-05
15	-1.482E-04	-9.980E-05	1.270E+01	1.531E-03	1.710E-03	25.4031	1.898E-05
16	-4.689E-04	-1.042E-03	1.270E+01	2.526E-03	3.669E-03	25.4051	4.072E-05
17	-1.119E-05	3.040E-04	1.270E+01	2.044E-03	2.348E-03	25.4041	6.373E-06
18	8.334E-05	-4.173E-04	1.270E+01	2.542E-03	2.967E-03	25.4051	8.052E-06
19	-1.101E-04	2.379E-05	1.270E+01	3.373E-03	3.486E-03	25.4067	9.459E-06
20	-1.237E-04	9.566E-06	1.270E+01	2.906E-03	3.030E-03	25.4058	8.223E-06
21	3.755E-04	2.013E-04	1.270E+01	3.360E-03	3.786E-03	25.4067	1.774E-05
22	-1.506E-04	9.654E-05	1.271E+01	5.121E-03	5.300E-03	25.4102	2.484E-05
23	-9.754E-04	-2.667E-04	1.270E+01	3.582E-03	4.594E-03	25.4072	2.153E-05
24	-3.654E-05	1.600E-04	1.270E+01	3.936E-03	4.100E-03	25.4079	1.922E-05

Note.  $\epsilon_0 = 0.000$  H= 50.80 mm R= 12.7 mm  $2\pi RH = 4053.66$

Multivariate regression analysis is used to find a fitted line, equation (6), for the cost coefficient:

$$K_p = a + b*\text{feed} + c*\text{speed} + d*\text{depth\_of\_cut} \quad (6)$$

Where  $a = 4.0833E-05$ ,  $b = -1.0659E-05$ ,  $c = -1.9555E-08$  and  $d = 4.3927E-06$

The above equation (6) could be used for estimating cost of manufacturing for various combinations of cutting speed, feed, and depth of cut.

## V. DISCUSSIONS AND CONCLUSION

The computed deviation parameters and cost coefficients are consistent with the speed, feed and depth of cut combinations. For example, for the samples 13, 14 (speed=1200 rpm, feed = 0.0508 mm/rev and depth of cut = 0.254 mm) the deviation parameters are comparable and the cost coefficients 2.478E-05 and 2.123E-05 are within measurement error limits. These experiments, thus, could establish deviation-based cost of manufacturing functions that could be used to estimate cost of manufacturing parts with geometric tolerances prior to actual manufacturing operations for specific operations on specific machines and at various combinations of cutting parameters. However, these cost functions have somewhat limited applicability as they are specific for machines, operations and operating parameter ranges. Further experiments need to be carried out with wide range of cutting parameter so

that these cost functions could be used for estimating cost of machining at different machining conditions.

With this experimental studies, the deviation parameters and cost coefficients have been established for cylindrical features. The authors plan to carry out similar studies to generate cost coefficients for machining other features like conical surfaces. Also, different sets of experiments would be needed to establish parameters for machining operations like milling, drilling, etc.

Instead of using the profilometer for measuring the surface deviations, an alternate approach for establishing the VCB could be to measure the surface deviations using a coordinate measuring machine (CMM) that could import the nominal shape of the work piece from a CAD model of the part and establish datums, VCB and AME by directly measuring the deviations of the machined surfaces/features.

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**Conflict of interest:** The authors declare that they have no conflict of interest.

**Ethical statement:** The authors declare that they have followed ethical responsibilities.

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## Appendix – 1: Measured Cylindrical Surface Deviation Data

### Measured Surface Deviations ( $\mu\text{m}$ )

Sample#	Pos	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2.60	2.58	2.40	2.48	2.20	2.06	2.22	2.28	2.36	2.08	2.68	2.52
	2	2.82	2.48	3.00	2.62	3.08	2.36	2.22	2.28	2.56	2.64	2.32	2.46
	3	3.06	3.48	3.68	3.44	3.60	3.40	3.24	3.18	2.76	2.94	2.86	2.92
2	1	4.86	4.62	4.64	5.00	4.40	4.38	4.16	3.80	3.76	4.24	4.46	4.62
	2	2.86	3.36	3.86	3.30	3.78	3.10	2.82	3.14	2.96	3.14	3.42	3.06
	3	3.34	3.36	3.62	3.60	3.18	3.06	3.70	3.40	3.06	3.20	3.64	3.38
3	1	3.56	3.96	3.96	3.78	3.84	3.88	3.68	3.60	3.64	3.88	3.32	3.44
	2	4.18	4.24	4.26	4.48	3.94	4.06	3.94	4.22	3.76	4.10	3.90	3.70
	3	3.52	3.58	3.06	3.50	3.32	3.44	3.06	3.16	3.32	3.50	4.12	3.80
4	1	3.80	3.98	4.30	3.66	3.78	4.16	3.96	3.86	4.10	4.60	4.16	4.34
	2	4.60	4.54	4.56	4.24	4.48	4.22	4.24	4.24	4.16	4.04	3.76	4.24
	3	5.26	4.90	5.12	4.40	4.56	4.48	4.50	4.24	4.84	4.48	4.40	5.02
5	1	2.62	3.00	2.64	2.66	2.60	2.98	2.88	2.86	3.10	2.92	2.94	3.04
	2	2.44	2.78	2.76	2.78	2.78	2.56	2.56	2.60	2.94	2.80	2.80	2.72
	3	2.82	2.76	2.84	2.78	3.10	2.88	2.54	3.04	2.74	2.74	2.60	2.52
6	1	3.62	3.46	3.56	3.56	3.44	3.48	3.42	3.20	3.34	3.32	3.30	3.44
	2	3.22	3.08	3.18	3.14	3.20	3.32	2.88	3.06	3.08	3.22	2.92	3.10
	3	3.16	3.24	3.28	3.36	3.38	3.32	3.14	3.04	3.44	3.28	3.46	3.58
7	1	2.28	2.46	2.50	2.84	3.16	3.16	2.64	2.18	2.54	2.88	2.80	2.58
	2	2.78	2.72	2.92	2.72	2.88	2.86	2.84	2.82	2.84	3.00	2.84	2.38
	3	2.64	2.52	2.36	2.52	2.64	2.88	2.66	3.34	3.20	2.72	2.82	3.04
8	1	2.94	3.06	2.84	2.82	2.96	2.68	3.16	2.74	2.78	3.22	3.16	2.90
	2	2.88	2.68	2.40	2.66	2.46	2.66	2.92	2.58	2.52	2.48	2.64	2.76
	3	2.94	2.24	2.32	2.40	2.26	2.20	2.24	2.14	2.70	2.56	2.56	2.42
9	1	2.96	2.86	3.18	3.06	3.28	3.10	3.06	3.26	3.04	3.14	3.52	3.06
	2	4.02	3.54	3.82	3.40	3.30	3.12	3.26	3.34	3.54	3.62	3.62	3.50

*Appendix: I Continued...*

	3	3.16	3.10	3.20	3.10	3.06	3.42	3.02	3.12	3.04	3.14	3.32	2.98
<b>10</b>	1	2.74	2.78	2.82	2.72	2.80	2.68	2.80	2.56	2.84	2.86	3.08	3.14
	2	2.16	2.16	2.30	2.30	2.12	2.04	2.48	2.32	2.38	2.14	2.26	2.22
	3	3.28	3.32	3.38	3.78	3.20	3.40	3.30	3.08	3.46	3.40	3.46	2.96
<b>11</b>	1	2.96	2.58	2.52	2.42	2.54	2.60	2.60	2.58	2.52	2.56	2.56	2.62
	2	2.96	2.86	3.06	2.58	2.70	2.92	2.82	2.84	2.74	3.04	3.04	2.96
	3	2.90	2.88	2.88	3.12	3.00	2.90	2.92	3.32	3.60	3.48	3.44	3.20
<b>12</b>	1	2.44	2.52	2.44	2.52	2.42	2.86	2.34	2.18	2.38	2.44	2.42	2.36
	2	2.40	2.38	2.16	2.26	2.44	2.32	2.20	2.14	2.40	2.68	2.34	2.52
	3	3.40	3.42	3.16	3.24	3.18	3.44	3.30	3.64	3.30	3.10	3.34	3.16
<b>13</b>	1	1.68	1.36	1.40	1.48	1.34	1.34	1.48	1.44	1.30	1.56	1.56	1.20
	2	1.80	1.82	1.74	1.74	1.70	1.72	1.68	1.74	1.74	1.80	1.78	1.76
	3	2.80	1.92	2.20	1.96	2.12	2.08	2.82	2.08	2.36	1.96	1.92	2.02
<b>14</b>	1	1.88	1.78	2.00	1.92	1.88	1.86	1.72	1.64	1.86	1.84	1.78	1.88
	2	1.16	1.18	1.14	1.10	1.34	1.28	1.36	1.34	1.40	1.14	1.24	1.20
	3	1.38	1.46	1.50	1.42	1.32	1.40	1.40	1.26	1.32	1.40	1.28	1.28
<b>15</b>	1	1.62	1.52	1.68	1.58	1.76	1.70	1.66	1.70	1.72	1.66	1.54	1.54
	2	1.44	1.42	1.44	1.42	1.36	1.34	1.38	1.26	1.32	1.32	1.32	1.30
	3	1.38	1.40	1.32	1.32	1.36	1.28	1.34	1.30	1.34	1.42	1.18	1.40
<b>16</b>	1	3.44	3.50	3.64	3.58	3.42	3.90	3.76	3.64	3.44	3.58	3.82	3.66
	2	2.60	2.48	2.46	2.50	2.40	2.34	2.34	2.26	2.22	2.28	2.28	2.46
	3	1.36	1.44	1.30	1.34	1.44	1.36	1.30	1.26	1.22	1.38	1.34	1.34
<b>17</b>	1	1.72	2.40	3.38	1.80	1.98	1.98	1.88	1.70	1.88	1.94	2.06	1.72
	2	1.58	1.40	1.64	1.70	1.58	1.44	1.60	1.56	1.64	1.70	1.72	1.56
	3	1.58	1.72	1.82	1.72	1.76	1.66	1.84	1.82	1.78	1.72	1.62	1.66
<b>18</b>	1	1.86	2.42	2.04	2.14	2.06	2.10	2.08	2.18	2.28	2.04	1.70	1.86
	2	1.60	1.58	1.76	1.70	1.70	1.78	1.96	1.70	1.98	1.62	1.66	1.52
	3	2.76	2.60	2.88	2.64	2.84	2.80	2.86	2.96	3.22	2.84	2.80	2.84
<b>19</b>	1	3.08	3.46	3.30	3.32	3.56	3.52	3.44	3.34	3.22	3.46	3.36	3.20

Appendix: I Continued...

	2	3.08	2.82	3.04	3.26	2.00	2.72	2.68	2.64	2.64	2.66	2.74	2.76
	3	3.34	3.06	3.42	3.30	3.08	3.02	3.20	3.04	2.90	3.00	3.22	3.32
<b>20</b>	1	2.64	2.50	2.28	2.46	2.34	2.42	2.38	2.72	2.86	3.00	2.64	2.58
	2	2.22	1.94	2.14	2.30	2.06	2.32	2.26	2.10	1.98	2.06	1.80	2.10
	3	2.54	2.92	3.04	2.90	3.14	2.62	3.16	2.80	2.78	2.80	2.70	2.46
<b>21</b>	1	2.60	2.30	2.52	2.68	2.68	2.58	3.10	3.10	2.90	2.66	2.64	2.52
	2	2.92	2.92	2.86	2.66	2.82	2.64	2.90	2.58	2.60	2.68	2.64	2.98
	3	3.78	3.88	3.86	3.58	3.60	3.62	3.58	3.40	3.82	3.84	3.70	3.58
<b>22</b>	1	4.04	3.78	3.40	3.74	3.74	3.62	3.82	3.70	3.72	3.22	3.70	3.84
	2	4.98	5.10	5.30	5.60	5.30	5.12	4.88	4.92	5.04	5.12	5.04	4.90
	3	5.02	4.64	4.56	4.60	4.58	4.64	4.70	4.86	5.10	4.86	4.82	4.70
<b>23</b>	1	4.20	4.54	4.36	4.50	4.38	4.52	5.02	4.12	4.16	4.38	4.26	4.44
	2	4.50	4.10	4.14	3.84	3.74	3.58	4.00	4.64	4.64	4.46	4.60	4.24
	3	2.66	2.52	2.22	2.08	2.12	2.24	1.94	2.22	2.38	2.54	2.26	2.66
<b>24</b>	1	3.50	3.36	4.00	4.72	3.84	4.26	3.96	3.44	3.92	3.74	3.44	3.60
	2	4.22	3.86	3.84	3.14	4.44	3.58	3.44	3.32	3.78	3.90	4.08	3.48
	3	3.40	3.86	3.82	4.14	3.76	3.56	3.74	4.08	3.64	3.98	3.32	3.14