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Software Reliability Model Based on Burr Type XII Distribution

[1]M.S. Ravikumar & [2]R.R.L. Kantam

[1]Department of Community Medicine, Konaseema Institute of Medical Sciences & Research Foundation/General Hospital, Amalapuram-533201, Andhra Pradesh, India

E-mail: *msrk.raama@gmail.com*

[2]Department of Statistics, Acharya Nagarjuna University, Nagarjunanagar-522510,

Guntur-Andhra Pradesh, India. E-mail: *kantam.rrl@gmail.com*

Abstract: A Non-Homogeneous Poisson Process (NHPP) with Burr Type XII Distribution based mean value function is considered and suggested as a software reliability growth model. Its parameters are estimated to measure the reliability of a developed software using some popular methods of estimation of statistical inference. The performance of the model with some existing similar models is studied through some measures of preference. The results are exemplified with the help of live data sets.

Keywords: Burr Type XII distribution, NHPP, Reliability estimation, Software Reliability Growth Model, Regression estimation

I. INTRODUCTION

Reliability is probability of failure free operation of any device. Software Reliability is probability of failure free performance of a software product. Causes for software failures are the latent bugs in it. A bug is suspected only if a failure occurs. When no failure occurs it does not mean that there are no bugs. A realised failure indicates presence of faults or bugs. The task is to search for the bugs and remove them. The process is called Debugging. It is reasonable to assume that an unknown finite number of bugs say "a" exist in the software. Let probability of finding a bug because of failure in an execution time of 't' be F(t) . Expected number of bugs to be traced before time't' is then $a.F(t)$. F(t)– a time dependent function can be considered as a Cumulative Distribution Function(CDF) of a random variable.

$a.F(t)$: The average number of failures experienced during time t.

$a.F(t)$ is called mean value function say m(t). A Poisson Process with $a.F(t)$ as the mean value function is called a Non-Homogeneous Poisson Process (NHPP). It is also called NHPP based Software Reliability Growth Model (SRGM). If the mean value function is of the form '$a.t$' the process becomes a Homogeneous Poisson Process. Consider [2] is one of the initial papers on NHPP based SRGM with F(t) taken as the cumulative distribution function of an exponential distribution.

In statistics, several CDF's exist. Each CDF would perhaps give a separate SRGM. Several authors considered a variety of NHPP based SRGMs ([3]). For considering [1] is one of the most recent papers focusing "Software Reliability Growth Models Incorporating Burr Type III Test- Effort and Cost-Reliability Analysis". Our paper considers the well-known Burr Type XII Distribution (BTXIID) as a model to represent the mean value function of an NHPP to develop a new SRGM. The mathematical treatment for developing the model and estimation method of software reliability are presented in Section II. The theoretical results of Section II are applied to a live data in Section III.

II. ESTIMATION OF SOFTWARE RELIABILITY

We know that the cumulative distribution function (cdf) and probability density function (pdf) of Burr Type XII Distribution are given by

$F(t) = 1 - (1 + t^c)^{-k}$ ---------------- (1)

$f(t) = ckt^{c-1}(1 + t^c)^{-(k+1)}$--------- (2)

Our mean value function would be

$m(t) = a[1 - (1 + t^c)^{-k}]$ ------------ (3)

Here a, c, k are the parameters of SRGM. The reliability of a software executed for 's' units of time to be failure free for an additional t-units of time is given by

$R(s|t) = e^{-[m(t+s)-m(t)]}$ ------------- (4)

For various forms of $m(t)$ we get different expressions for Software Reliability. Knowledge about Software Reliability speaks of Software Quality. From the point of view statistical science software reliability is a function of SRGM parameters, the used time, and the future required trouble free time. If the parameters of SRGM are known Software Reliability is known. If the parameters are unknown Software Reliability requires Estimation.

Statistics provides different admissible methods of estimation based on supplied data on Software failures. This is the interface between Software Engineering and Statistical Science. Under Ideal conditions adoption of a classical method of estimation for the parameters and developing the software reliability is suggested. Our model is a Burr XII Type based NHPP and the SRGM.

The NHPP is $\frac{e^{-m(t)}[m(t)]^r}{r!}$. ------------ (5)

Here m(t) is the mean value function of the Poisson Distribution. We take Burr Type XII distribtuion CDF multiplied by 'a' as m(t) . We require data which is generally in two types

(i) Failure Count Data.

(ii) Inter Failure Time Data.

In this paper we develop methodology given for failure count data. The basic equations to find the parameters by the well-known maximum likelihood method are

$LLF = \sum_{i=1}^{n}(y_i - y_{i-1}).log[m(t_i) - m(t_{i-1})] - m(t_n)$ --------------- (6)

$$0 = \sum_{i=1}^{n} \frac{\frac{\partial}{\partial\theta}m(t_i) - \frac{\partial}{\partial\theta}m(t_{i-1})}{m(t_i) - m(t_{i-1})}(y_i - y_{i-1}) - \frac{\partial}{\partial\theta}m(t_n)$$

Here y_i stands for the cumulative number of failures experienced up to execution time t_i. For our model the equations to get the estimate of the parameters after simplification are

$\frac{\partial \ln L}{\partial c} = 0 \Rightarrow \frac{n}{c} + \sum_{i=1}^{n}\left[\frac{y_i}{y_i^c} - \frac{k}{(1+y_i^c)^{(k+1)}}\right] = 0.$ -------------- (7)

$\frac{\partial \ln L}{\partial k} = 0 \Rightarrow \left[\frac{n}{k} + \sum_{i=1}^{n}\frac{-c}{(1+y_i^c)^{(k+1)}}\right] = 0.$ -------------------- (8)

Use (7) and (8) for iterative solutions of these two would give maximum likelihood estimation. In view of the mathematical form of the CDF of BTXII distribution we suggested a non-conventional approach namely regression method to estimate the model parameters which in turn are used to get estimate of software reliability. We suggest a regression approach method which is simpler and effective for estimation. It is based on a linear representation of successive differences for the failure rate. Adoptability of our model in relation to the standard model is verified with the help of an error criterion. The following are the steps:

- Identify a failure count data: y_i, t_i.
- Estimate the parameters a, c, k of the SRGM.
- Use the estimates along with t_i in the mean value function to get the expected number of failures upto time t_i say $\hat{y}_i$.
- Find the $\hat{y}_i$ called the error.
- Find the maximum of $\hat{y}_i$ called the maximum possible error associated with our model.
- Choose a standard model as SRGM that already exists in literature we have taken the one based on exponential distribution. This is also called the Goel and Okumoto model.
- Its mean value function is $m(t) = a[1 - e^{-bt}]$ Okumoto model.
- Compare the maximum errors for the same data with the two different models.
- Prefer that model for which the maximum error is less.
- We have identified a failure count data given in [3] Software Reliability as given below.

III. ILLUSTRATION

Failure data in a one-hour interval and number of failures.

Table 1. Failure data in a one-hour interval

Hour	***Number of failures***	***Cumulative failures***	***Hour***	***Number of failures***	***Cumulative failures***
1	27	27	14	5	111
2	16	43	15	5	116
3	11	54	16	6	122
4	10	64	17	0	122
5	11	75	18	5	127
6	7	82	19	1	128
7	2	84	20	1	129
8	5	89	21	2	131
9	3	92	22	1	132
10	1	93	23	2	134
11	4	97	24	1	135
12	7	104	25	1	136
13	2	106			

For our model

$$\hat{a} = 95$$

$$\hat{c} = 6.99092$$

$$\hat{k} = 2.702603$$

For our model the maximum of the error measured through the absolute deviation of the observed failure time y_i and the corresponding estimated failure time $\hat{y}_i$, is found to be maximum of $|y_i - \hat{y}_i| = 53.40651$.

For Goel and Okumoto model the mean value function and the NHPP are

$m(t) = a[1 - e^{-bt}]$ and $\frac{e^{-m(t)}[m(t)]^r}{r!}$.

Parameter estimates of the model for a data are

$$\hat{a} = 138.3779$$

$$\hat{b} = 0.1334$$

and maximum of $|y_i - \hat{y}_i| = 136$.

Since error associated with BTXII SRGM is less than that of Goel and Okumoto SRGM, we say that our SRGM is a better model than Goel and Okumoto model. Hence, we conclude for this example our model is more suitable.

Conflict of Interest: The authors declare that they have no conflict of interest

Ethical Statement: The authors declare that they have followed ethical responsibilities

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