Numerical Solutions of One-Dimensional Bioheat Transfer Equation in Cylindrical Living Tissues

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Abstract: One-dimensional steady state bio-heat transfer model based on pennes' for the temperature distribution in cylindrical living tissue is disscused. The obtained solution is applied to analyze the effects of the metabolic heat generation, the tissue thermal conductivity, the blood perfusion and the coefficient of heat transfer on the temperature distribution in living tissues. In this paper we use the Finite Difference Method and Galerkin Finite element method to find the results of the model and we compare the obtained result by numerical techniques . The result obtained suggested that the solution in this paper is useful to easily and accurately study the thermal behavior of the biological system and can be extended to thermal behavior research of biological system, thermal parameter measurements, temperature field reconstruction and clinical treatment.

Keywords: Pennes' Bio-heat transfer equation, Finite Difference Method,Galerkin Finite Element Method

I. INTRODUCTION

A human body behaves under different environmental conditions of air temperature, humidity and wind velocity. Gurung [1,3,4,5] and Gyton [6] studied the transport of heat in living tissue which is also a complex process involving multiple mechanisms in tissue, including conduction, convection, radiation, metabolism, evaporation and phase change . The normal body core temperature is about 37 ^{0}C. The maintenance of body temperature is a dynamic system. If heat is greater than heat production then the body core temperature drops. Likewise if heat loss is less than heat production then the core temperature rises. So, the rise or drop in core temperature is equally dangerous, so body temperatures are kept constant.

Recently research development shows that the heat transfer problem in biological tissues becomes one of complicated issues. But there are several discussion an finding in this field. In 1948 Hennery Pennes [9] proposed a simple linear mathematical model for describing the thermal interaction between human tissues and perfused blood, and effect of metabolism. He measured the radial temperature in the forearm by pulling fine thermocouples through the arms of nine recumbent subjects. That was based on experimental observation. Using this experimental concept Mitchell and Myers Model [15],Keller and Seiler Model [16], Wulff Model [17], Chen and Holmes model [18], Weinbaum and Jiji Model [19], Khaled and Vafai Model [20],Nakayama and Kuwaha Model [21] has discussed the thermal behavior in various parameter constant with human layers of tissues. The temperature distributation take over in blood and artery tissues level. Cooper and Trezek [2]found an analytic solution of heat diffusion equation for brain tissue with negligible effect of blood flow and metabolic heat generation. D. B. Gurung [3] studied the abnormal thermoregulation model in human dermal part. He also studied

the steady state and unsteady state temperature distribution in three layers of dermal part. And used the numerical method for the purposed of the solution techniques which is most accurate for the analysis of the temperature in human body.

In this paper, we study the effect of thermal parameters of dermal part in cylindrical living tissue. The metabolic heat generation and rate of blood perfusion is taken uniformly in dermal part of living tissue. The linear function is considered. The outer surface of the body is exposed to the environment and the loss of heat from the skin surface is assumed due to convection and radiation. Here, we neglected the axial and angular direction and considered only the radial direction steady state model. The numerical result(Finite Difference Method and Galerkin Finite Element Method, Fanisam[22]) obtained is exhibited graphically and compared with the result of Zhang et al.[12] by applying the suitable values of physical and physiological parameters. The solution obtained can be used for the measurement of thermal parameters, reconstruction of the temperature field and thermal diagnosis and in the treatment that maximizes the therapeutic effect while minimizing unwanted side effect. It may also be useful to design medical devices to perform within a special range of temperature rate of heating and cooling as Zhang et al. [12].

II. MODEL FORMULATION

Many bio-heat equations are developed from the studies of blood perfusion process in human body. The mathematical model used for bio-heat transfer is based on Pennes' equation. The Pennes' model[9] is preferable for the study of heat transfer between blood and tissue which also associates the effect of metabolism and blood perfusion. The modified Pennes' equation is written as

$$\rho c\frac{\partial T}{\partial t}=\nabla.(k\nabla T)+w_b c_b(T_a-T)+q_m \qquad (1)$$

where, ρ, c, k are the density(Kg/m^3), the specific heat(J/Kg.^{0}C) and the thermal conductivity of tissue(W/m.^{0}C) respectively.w_b is the blood perfusion rate per unit volume(Kg/s.m^3), c_b is the specific heat of blood(Kg/m^3), q_m is the metabolic heat generation per unit volume(W/m^3), T_a represents the temperature of arterial blood(^{0}C) and T is the tissue temperature(^{0}C). The one dimensional steady state bio heat equation of cylindrical living tissue is given by

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)+M(T_a-T)+q_m=0 \qquad (2)$$

with the boundary conditions $r=0,\ \frac{dT}{dr}=0$, and $r=R,\ -k\frac{\partial T}{\partial r}=h_A(T-T_\infty)$

where R is the radius of concerned tissue, h_A heat transfer coefficient,and T_∞ is ambient temperature.

III. SOLUTION TECHNIQES

A. FINITE DIFFERENCE METHOD

From equation (2), we have

$$\frac{d^2R}{dr^2}+\frac{1}{r}\left(\frac{dT}{dr}\right)+\frac{M}{k}(T_a-T)+\frac{q_m}{k}=0 \qquad (3)$$

Using the Finite Difference Method (FDM) [7,8,11,13] the following can be obtained.

$$\text{for } i=0;\ 4T_1-\left(4+\frac{h^2M}{k}\right)T_0+F=0 \quad \text{where} \quad F=\frac{h^2}{k}\left(MT_a+q_m\right) \qquad (4)$$

$$\text{for } i=1,2,...,R-1; \quad \left(1-\frac{1}{2i}\right)T_{i-1}+\left(1+\frac{1}{2i}\right)T_{i+1}-\left(2+\frac{h^2M}{k}\right)T_i+F=0 \quad (7)$$

$$\text{and for } i=R;\ 2T_{R-1}-DT_R+E+F=0 \quad (5)$$

$$\text{where, } D=\left[\left(2+\frac{Mh^2}{k}\right)+\left(1+\frac{1}{2R}\right)\frac{2hh_A}{k}\right],\ E=\left(1+\frac{1}{2R}\right)\frac{2hh_AT_\infty}{k}$$

From equations (3), (4) and (5) we can find the following system of linear equations represented in matrix form; $AX=B$ (6)

where,

$$A=\begin{bmatrix} -\left(4+\frac{h^2M}{k}\right) & 0 & 0 & \cdots & 0 \\ 0.5 & -\left(2+\frac{h^2M}{k}\right) & 1.5 & \cdots & 0 \\ 0 & 0.75 & -\left(2+\frac{h^2M}{k}\right) & 1.25 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 2 & -D \end{bmatrix},\ X=\begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ \vdots \\ T_R \end{bmatrix} \text{ and } B=\begin{bmatrix} -F \\ -F \\ -F \\ \vdots \\ -E-F \end{bmatrix}$$

The matrix (6) gives the nodal values in FDM which enables to compute the temperature distribution profile in dermal part.

B. GALERKIN FINITE ELEMENT METHOD

Using weak Formulation of Galerkin Finite Element Methods(FEM)[7,10] with $r_a=0$ and $r_b=R$ in the equation (2), we get

$$\int_{r_a}^{r_b} w\left[\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)+M(T_a-T)+q_m\right]dr=0 \quad (7)$$

putting $a=kr$ and Integrating equation (7) we get

$$\int_{r_a}^{r_b}\left[w\frac{d}{dr}\left(a\frac{dT}{dr}\right)+Mwr(T_a-T)+q_m\,wr\right]dr=0 \quad (8)$$

After simplified and using the trial function into equation(8) we get

$$\left[K^e\right]\left\{T^e\right\}=\left\{f^e\right\}+\left\{Q^e\right\} \quad (9)$$

where, $K_{ij}^e=\int_{r_a}^{r_b}\left[a\frac{d\psi_i^e}{dr}\frac{d\psi_j^e}{dr}-M\psi_i^e\psi_j^e r\right]dr$, $f_i^e=\int\psi_i^e f r dr$, $f=MT_a+q_m$

Let the linear function be $\psi_1^e=\frac{r_b-r}{h_e}$ and $\psi_2^e=\frac{r-r_a}{h_e}$. Using explicit form $a=a_e r,\ f=f_e$

and $r = r_a + \bar{r}$. Thus we get

$$K_{11}^e = \frac{a_e}{h_e}\left(r_a + \frac{h_e}{2}\right) + \frac{Mh_e}{12}(4r_a + h_e), \quad K_{12}^e = -\frac{a_e}{h_e}\left(r_a + \frac{h_e}{2}\right) + \frac{Mh_e}{12}(2r_a + h_e)$$

$$K_{12}^e = K_{21}^e, \quad K_{22}^e = \frac{a_e}{h_e}\left(r_a + \frac{h_e}{2}\right) + \frac{Mh_e}{12}(4r_a + 3h_e), \; f_1^e = \frac{f_e h_e}{6}(3r_a + h_e), \; f_2^e = \frac{f_e h_e}{6}(3r_a + 2h_e)$$

$$\text{Thus, } \left[K^e\right] = \frac{a_e}{h_e}\left(r_a + \frac{h_e}{2}\right)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{Mh_e}{12}\begin{bmatrix} 4r_a + h_e & 2r_a + h_e \\ 2r_a + h_e & 4r_a + 3h_e \end{bmatrix} \text{ and } \left\{f^e\right\} = \frac{f_e h_e}{6}\begin{bmatrix} 3r_a + h_e \\ 3r_a + 2h_e \end{bmatrix} \quad (10)$$

From the equation(10) we find the element at $r_a = 0, h, 2h, ..., nh$ and can represent the elements in the matrix form; $PX = Q$ (11)

$$\text{where, } P = \begin{bmatrix} a & c & 0 & \cdots & 0 \\ c & 4b & 3c & \cdots & 0 \\ 0 & 3c & 8b & 5c & 0 \\ 0 & 0 & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & p \end{bmatrix}, X = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ \vdots \\ T_n \end{bmatrix} \text{ and } Q = N\begin{bmatrix} 1 \\ 6 \\ 12 \\ \vdots \\ v_1 \end{bmatrix} + \begin{bmatrix} Q_1^1 \\ Q_1^2 + Q_2^1 \\ Q_1^3 + Q_2^2 \\ \vdots \\ Q_2^n \end{bmatrix}$$

$$\text{and } a = \frac{6K + Mh^2}{12}, \; b = \frac{6K + 2Mh^2}{12}, \; c = \frac{-6K + Mh^2}{12}, \; p = 6K(2n-1) + ((4n-1)Mh^2), \; N = \frac{fh^2}{6}, \; v_1 = (3n-1)$$

For the boundary condition of extreme points of each linear element, we have the following

$$P = \begin{bmatrix} a & c & 0 & \cdots & 0 \\ c & 4b & 3c & \cdots & 0 \\ 0 & 3c & 8b & 5c & 0 \\ 0 & 0 & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \alpha \end{bmatrix}, X = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ \vdots \\ T_n \end{bmatrix} \text{ and } Q = N\begin{bmatrix} 1 \\ 6 \\ 12 \\ \vdots \\ v_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \beta \end{bmatrix}$$

where, $\alpha = p + Rh_A T_{n+1}, \beta = Rh_A T_\infty$.

Hence, the system of equations (11) gives the nodal values in Galerkin FEM. These nodal values are further used to find the temperature distribution profiles in dermal part.

IV. RESULTS AND DISCUSSION

In this section, the effect of thermal conductivity and heat transfer coefficients in living tissue using the Finite Difference solution equation (6), and Galerkin Finite Element solution equation (11) will be discussed. For the comparative study of numerical results of these effects based on the discussed solution techniques, we consider the following parameter values given in Zhang et al. [12] with normal ambient temperature(T_∞) 25 ^{0}C and number of nodes more than 30.

Table 1: Values of Parameters for Theoretical Analysis.

w_b Kg /s.m^3	c_b J/Kg.^{0} C	K W/ m.^{0}C	h_A W/ m^2.^{0}C	q_m W/m^3	T_a ^{0}C	R m
3	3850	0.48	30.023	1085	37	0.0285

A. Effects of the Thermal Conductivity

The various value of thermal conductivity of dermal part are taken as 0.24 W/m.^{0}C, 0.48 W/m.^{0}C and 0.72 W/m.^{0}C for the observation of the thermal conductivity effects in living tissue. Figure(1) represent the graph of thickness verses body temperature obtained by using FDM solution and FEM solution respectively.

(a) FDM Solution. (b) FEM Solution.

Figure 1: Effects of the Thermal Conductivity.

The observed results in Figure(1)shows that the temperature distribution in living tissue is decreasing smoothly and then temperature falls sharply nearly the skin surface due to the conduction process at the outer surface of the living tissue. The value of temperature at any point of dermal part near core at high thermal conductivity is less than the temperature at low thermal conductivity. The results obtained from Figure(1) exhibit approximately the same value of temperature distribution at a given thickness of dermal part measured from the bodycore. Hence the FDM and FEM are suitable numerical approximation techniques for the study of thermal conductivity effect in dermal region.

B. Effects of the Heat Transfer Coefficient

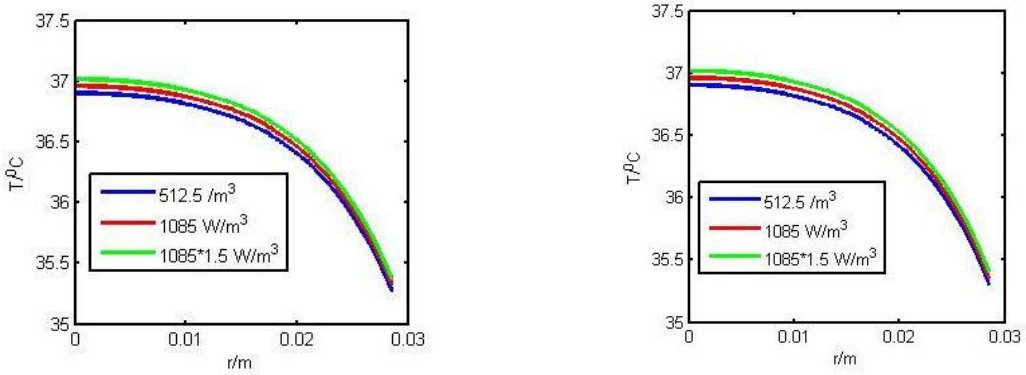
(a) FDM Solution. (b) FEM Solution.

Figure 2: Effects of the Heat Transfer Coefficient.

The value of parameter 5.023W/m^2.^{0}C, 10.023W/m^2.^{0}C and 20.023W/m^2.^{0}C are used to the study for the effects of heat transfer coefficient in the dermal part of livingtissue.The solution graphs in the FDM and FEM are presented in the Figure(2) respectively. The graphs in Figure(2) shows the decrease of tissue temperature very sharply near the skin surface with the increase of heat transfer coefficient.The rapid falling of temperature near skin surface at high heat transfer coefficient rate is due to the direct contact of skin surface with environment because heat transfer rate contribute to transfer of heat at skin surface. Comparing the graphs obtained in Figure(2) we observed no notable differences in the results of temperature distribution model. So, numerical approximation techniques best represent the solution techniques for the study of heat transfer coefficient in the model.

C. Effects of the Metabolic Heat Generation

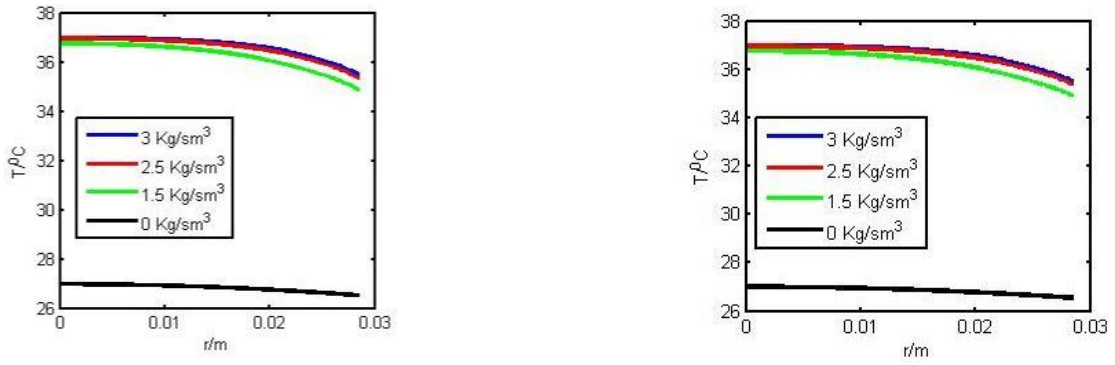
The effects of metabolic heat generation are observed with the given values 512.5W/m^3,1085W/m^3, and 1085×1.5 W/m^3 of dermal part of living tissue. Fig.3 (a), (b) show the graphs of temperature distribution profiles in living tissue using FDM and FEM. Anatomically, the concentration of blood vessels towards the skin surface is less and hence there is negligible effects of metabolism toward skin surface. This fact is clearly ex-hibited in the temperature profiles of Fig.3 so the significant difference of temperature distribution in skin surface in not observed. The dermal temperature towards the core increases with the increase of metabolic rate, because metabolic rate generates heat. The same results for metabolic effects are observed in the discussed three solution methods.



(a) FDM Solution. (b) FEM Solution.

Figure 3: Effects of the Metabolic Heat Generation

D. Effects of the Blood Perfusion



(a) FDM Solution. (b) FEM Solution.

Figure 4: Effects of the Blood Perfusion

The different value of parameter taken for discussion of the effect of blood perfusion rates are 3Kg/sm^3, 2.5Kg/sm^3, 1.5 Kg/sm^3, and 0Kg/sm^3. The temperature distribution for perfusion effects at various perfusion rate are obtained by using FDM solution and FEM solution respectively. From the graphs in the Fig.4 we observed that blood perfusion play a significant role in temperature distribution model of living tissue. The Fig.4 (a),(b) also represent the increase of tissue temperature with the increase of perfusion rate. This is due to the high rate of heat distribution caused by blood perfusion. Comparing the graphs obtained in Fig.4 (a),(b) we observed no notable differences in the results of temperature distribution model. So, numerical approximation techniques best represent the solution techniques for the study of blood perfusion effects in the model.

V. CONCLUSION

In this study we use the different values of thermal conductivity, metabolic heat generation, blood perfudion and heat transfer coefficient of the dermal part of cylindrical living tissue. The effects of different thermal parameters are compared by using the solution techniques FDM solution and FEM solution in the cylindrical bio-heat equation.From the comparison, we are taking constant parametric values for the solution of the model. The solution of the model obtained using numerical solution techniques coincide exactly with analytic results.In reality dermal part is a complex composite of tissue having various layers. These layers have different parametric value and someparametric value may be considered reasonably as a function of thickness.Under these consideration, it is very complex or sometime not possible to find the analytic solution. Due to this reason, this comparative study makes more clear for researchers for the selection of best approximation numerical tool.

Conflict of interest: The authors declare that they have no conflict of interest.

Ethical statement: The authors declare that they have followed ethical responsibilities.

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This volume is dedicated to Late Sh. Ram Singh Phanden, father of Dr. Rakesh Kumar Phanden.

