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Testing of Hypothesis based on Rayleigh versus Gamma Models

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***Abstract*:** Two popular models Rayleigh and gamma (2) distributions are considered to verify whether one can be an alternative to the other. The cumulative distribution function of gamma (2) is not analytically tractable, whereas for Rayleigh distribution is tractable which motivated for the study. Test statistics based on likelihood ratio is suggested to discriminate between Rayleigh and gamma (2) models. The percentiles and power of the proposed test statistics were also tabulated, and a comparison was made with respect to the power for a given sample and level of significance.

Keywords: Rayleigh distribution, gamma (2) distribution, Likelihood Ratio Criterion.

I. INTRODUCTION

The cumulative distribution function and probability density function of Rayleigh distribution (Weibull with shape parameter 2), gamma distribution with shape parameter 2 and scale parameter σ are given by:

$$F_0(x) = 1 - e^{\left(\frac{-x^2}{2\sigma^2}\right)}; 0 \le x \le \infty, \sigma > 0 \tag{1.1}$$

$$f_0(x) = \frac{x}{\sigma^2} e^{\left(\frac{-x^2}{2\sigma^2}\right)}; 0 \le x \le \infty, \sigma > 0 \tag{1.2}$$

$$F_1(x) = 1 - e^{\left(\frac{-x}{\sigma}\right)}\left(1 + \frac{x}{\sigma}\right); x \ge 0, \sigma > 0 \tag{1.3}$$

$$f_1(x) = \frac{x}{\sigma^2} e^{\left(\frac{-x}{\sigma}\right)}; x \ge 0, \sigma > 0 \tag{1.4}$$

The frequency curve of the two distributions look alike and cumulative distribution of Rayleigh distribution is analytically invertible and that of a gamma distribution is an incomplete gamma function which is extensively tabulated in [10]. The graph of the frequency curve of Rayleigh distribution and gamma distribution with shape parameter 2 are given in figures 1.1 and 1.2.

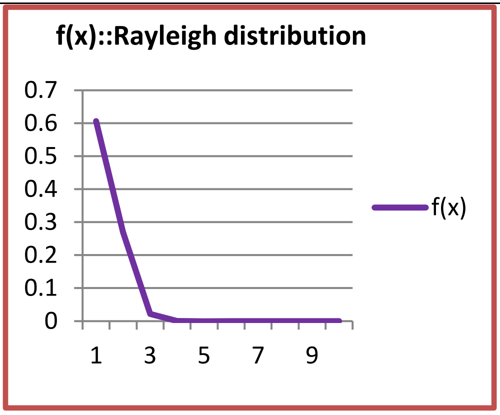

Figure – 1.1

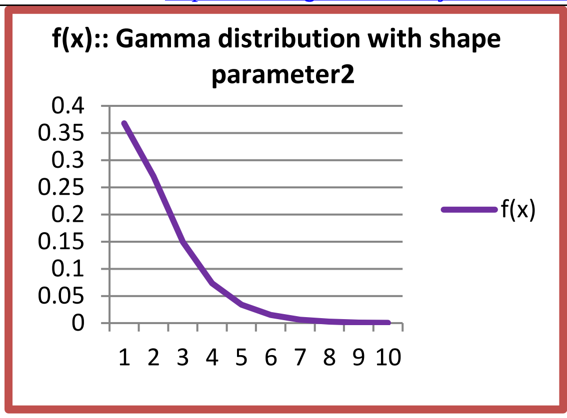

Figure – 1.2

The discrimination between Rayleigh distribution look similar to gamma distribution with shape parameter 2 is studied in this paper. Some of the authors working in this direction are [5] for the Weibull and the generalized exponential distributions, [6] for the gamma and generalized exponential distributions, [13] for the Weibull and log-normal distributions, [11] for normal and Laplace distributions, [12] for log-normal and generalized exponential distributions. For the gamma**,** log-normal and the generalized exponential distributions, [14] for the gamma and log-normal distributions, [15] for the generalized Rayleigh and log-normal distributions, [1] for log-normal, Weibull and generalized exponential distributions, [2] for the log-normal and log-logistic distributions, [3] for the Weibull and log-normal distributions for type-II censored data, [4] for bivariate generalized exponential and bivariate Weibull distributions, [17] for generalized exponential distributions and some life test models based on population quantiles, [19] for half normal and half logistic distributions, [18] for log-logistic and Rayleigh distributions, [7] for likelihood test for linear failure rate and exponential distributions, [8] for linear failure rate and Rayleigh distributions, [9] for Burr Type XII distribution and Weibull-exponential distribution, [16] for Burr type XII and Weibull-exponential distributions. Some inferential procedures are derived in Inverse Burr type X distribution, [21]. The test of the economic sampling plans for Burr type X distribution for percentile calculations is in [20]. In this paper, we study in that direction through likelihood ratio criterion, which is narrated and the simulation results are presented in Section 2. The results are concluded in Section 3.

II. LIKELIHOOD RATIO CRITERION

Rayleigh distribution is taken as null population and gamma 2 is considered as an alternative population. i.e., H_o: A given sample $(x_1, x_2, \ldots, x_n)$ belongs to Rayleigh Model
H_1: The sample $(x_1, x_2, \ldots, x_n)$ belongs to gamma (2) Model
The MLE of σ for Rayleigh distribution is given by Equation (2.1)

$$\frac{\partial LogL}{\partial \sigma} = 0 \Longrightarrow \hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{2n}} \qquad (2.1)$$

Similarly, MLE of σ for gamma 2 given by Equation (2.2)

$$\frac{\partial LogL}{\partial \sigma} = 0 \Longrightarrow \hat{\sigma} = \frac{\bar{x}}{2} \qquad (2.2)$$

The likelihood ratio statistic is defined as $\lambda = Ln\left(\frac{L_1}{L_0}\right)$evaluated at the respective MLEs of σ.

$$i.e., \lambda = nlog\,(2) + nlog(\textstyle\sum_{i=1}^{n} x_i^2) - 2nlog(\bar{x}) - n\log(n) \qquad (2.3)$$

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The ratio $\lambda = \frac{L_1}{L_0}$ given by equation (2.3) represents the ratio of the likelihood of L_1 to that of L_0 with a sample drawn from L_0. Hence, $\frac{L_1}{L_0}$ is the ratio of a smallest probability to a larger probability and is expected to be small. Therefore, the null hypothesis H_o: The sample drawn from the Rayleigh can be tested using the percentiles of the likelihood ratio $\frac{L_1}{L_0}$, thus the statistic $\lambda = \frac{L_1}{L_0}$ can be taken as test statistic to test the null hypothesis. Since the distribution of $\frac{L_1}{L_0}$ is not analytically tractable and we have computed the percentiles of empirical sampling distributions $\lambda = \frac{L_1}{L_0}$ with the help of 10,000 simulation runs of sample sizes ***n*** = 2(1)10,15,20,25 are given in Table 2.1, and the corresponding selected acceptable values from alternative probabilities are presented in Table 2.2.

Table-2.1: Likelihood Ratio Criterion of Rayleigh Distribution Vs Gamma 2 Percentiles

n	**Percentiles**					
	0.9985	**0.995**	**0.99**	**0.975**	**0.99**	**0.00135**
2	1.3735	1.1908	1.0478	0.8821	0.7374	0.0071
3	1.3536	1.0978	0.9005	0.7095	0.5795	0.0028
4	1.3145	0.8870	0.7385	0.5441	0.4061	0.0016
5	1.1587	0.7783	0.5868	0.4188	0.3059	0.0005
6	0.8008	0.5258	0.4024	0.2949	0.2148	0.0002
7	0.6788	0.4228	0.3238	0.2074	0.1414	0.0001
8	0.5369	0.3029	0.2303	0.1465	0.0993	5.08E-05
9	0.5389	0.2542	0.1726	0.1130	0.0699	2.57E-05
10	0.3384	0.1786	0.1236	0.0727	0.0468	0.00001
15	0.0829	0.0355	0.0202	0.0102	0.0059	2.49E-07
20	0.0115	0.0048	0.0028	0.0013	0.0006	5.31E-09
25	0.0018	0.0006	0.0004	0.0002	0.0007	3.34E-10

Table-2.2: Acceptable Values from Alternative Probabilities

n	**Percentiles**		
	90%	**95%**	**99%**
2	0.4800	0.5552	0.6841
3	0.3773	0.4612	0.5948
4	0.2850	0.3565	0.5019
5	0.2370	0.3034	0.4361
6	0.1988	0.2529	0.3560
7	0.1477	0.1974	0.3189
8	0.1269	0.1729	0.2770
9	0.0984	0.1422	0.2326
10	0.0802	0.1124	0.1951
15	0.0262	0.0425	0.0874
20	0.0088	0.0158	0.0372
25	0.0025	0.0055	0.0144

The power of the test statistic λ is also tabulated for three levels (10%, 5%, 1%) at sample sizes ***n***= 2(1)10, 15, 20, 25 by simulating samples from H_1 and using the values of λ. The count of λ value that fall beyond the table value of Table 2.1 shall speak of the power of the test λ. These are presented in Table 2.3.

Table-2.3: Power of the test λ

n	**90%**	**95%**	**99%**
2	0.5200	0.4448	0.3159
3	0.6227	0.5388	0.4052
4	0.7150	0.6435	0.4981
5	0.7630	0.6966	0.5639
6	0.8012	0.7471	0.6440
7	0.8523	0.8026	0.6811
8	0.8731	0.8271	0.7230
9	0.9016	0.8578	0.7674
10	0.9198	0.8876	0.8049
15	0.9738	0.9575	0.9126
20	0.9917	0.9842	0.9628
25	0.9975	0.9945	0.9856

These tables indicate that even with the help of a small sample of size ***n***=2 and above, the power remains to be increasing as ***n*** increases. Therefore, the statistic λ proposed in this section can be discriminated between the null and alternative populations with a high-power value as given in Table 2.3.

III. CONCLUSION

We therefore conclude that for samples $n < 5$, Rayleigh and gamma (2) make a distinction and for $n \geq 5$, Rayleigh and gamma (2) are equally likely models for a sample generated from either Rayleigh or gamma (2) models.

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